Secondary Mathematics Teacher Beliefs About the Learning and Teaching of Mathematics[1]

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This paper continues an ongoing research investigation of teacher beliefs towards the learning and teaching of mathematics. The focus is the espoused beliefs of a group of secondary teachers. Analysis of returned questionnaires allows secondary teachers to be profiled according to correlations of their espoused beliefs about mathematics, mathematics learning and mathematics teaching, along a continuum of learning and teaching approaches.

Introduction

Though the investigation of teachers' beliefs is a relative new area of research (Thompson, 1992), evidence suggests that such beliefs play a critical role in determining how teachers teach (Pajares, 1992; Weissglass, 1992). This report continues the research agenda presented at MERGA 17 (Perry & Howard, 1994) which set the basis for the reported investigation of primary teacher's use of manipulatives (Howard, Perry & Conroy, 1995), primary teacher's espoused beliefs about the learning and teaching of mathematics (Perry, Howard & Conroy, 1996) and secondary teachers' use of manipulatives (Howard, Perry & Lindsay, 1996).

This investigation of the espoused beliefs of 249 secondary mathematics teachers, working during 1996 in government and Catholic high schools in the south western suburbs of Sydney, adds to the literature and provides some basis from which decisions concerning mathematics syllabus and pedagogy might be made. Further, it provides a foundation on which interviews and classroom observations may be based in order for comparisons to be made between secondary mathematics teachers' espoused and enacted beliefs.

Teacher Beliefs about the Learning and Teaching of Mathematics

The development of one's beliefs about the nature of mathematics and how one does mathematics "*are important not only because they influence how one thinks about, approaches, and follows through on mathematical tasks but also because they influence how one studies mathematics and how and when one attends to mathematics instruction*" (Garofalo 1989, p. 502). It is recognised that a student's prime source of mathematical experiences is the classroom (Frank, 1988) and what occurs in the mathematics classroom influences student beliefs. Critical to the classroom implementation of the learning and teaching of mathematics is the teacher and, in particular, the beliefs of the teacher. All teachers hold beliefs towards the learning and teaching of mathematics. These beliefs influence and guide teachers in their decision making and implementation of teaching strategies (Baroody, 1987). Indeed, it has been suggested that the investigation of peoples' beliefs about learning and teaching may well be the most critical factor in educational research (Pajares, 1992).

The beliefs espoused by teachers of mathematics fall within a continuum ranging from a traditional view of mathematics being taught to students through a teacher transmission of mathematical skills and knowledge to one where students are actively involved with mathematics through "*constructing their own meaning as they are confronted with learning experiences which build on and challenge existing knowledge*" (Anderson 1996, p. 31).

One model for categorising beliefs about the teaching of mathematics presented by Kuhs and Ball (1986) suggests that teachers can hold views that fall into four broad categories: learner focused; content focused with an emphasis on conceptual knowledge; content focus with an emphasis on performance; classroom focused. A further model, categorising teachers'

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espoused beliefs on the continuum of transmission to child-centred, has been used in an attempt to profile the mathematical beliefs held by primary school teachers (Perry, Howard & Conroy, 1996). In this approach, conflicting beliefs may become apparent when attempting to cluster and categorise espoused beliefs, partly because teachers will hold beliefs along the continuum and across identified views. Thompson (1992) has reported that teachers' conceptions of mathematics appear to be related to differences in their views about teaching mathematics. Teachers' beliefs seem to evolve from their teaching experience rather than formal study and there appears to be a strong relationship between teachers' conceptions of teaching and their conceptions of students' mathematical knowledge. This study focuses on identifying and categorising the espoused beliefs of secondary mathematics teachers towards the nature of mathematics, mathematics learning and mathematics teaching.

Methodology

The data for this investigation were collected using a specifically designed questionnaire consisting of both multiple choice and open-ended questions covering the following areas:

1. subject demographics such as gender, age, position in school, nature of teacher training, length of teaching experience, class(es) currently taught, class size, classes taught over the last ten years;
2. use of manipulatives in mathematics learning and teaching such as which are used, why and how they are used, and the areas of mathematics in which they are used;
3. beliefs about mathematics, mathematics learning and mathematics teaching.

The questionnaire relies on the self reporting of the teachers and parallels much of the work reported in Hatfield (1994). It is substantially the same questionnaire which was used to gather data from primary school teachers (Perry, Howard & Conroy, 1996).

In September, 1996, the questionnaire was posted, with reply paid envelopes, to 52 secondary schools in the South Western suburbs of Sydney. Fifteen schools were part of the Catholic Secondary Schools System while 37 were NSW Department of School Education (government) schools. Schools were contacted by telephone to gain the Principals' initial approval to undertake the survey in the schools and to ascertain the number of mathematics teachers in each school. Sufficient numbers of the questionnaire were posted to cover all mathematics teachers at each school. Two hundred and forty-nine responses were received by mid-October, 1996. Data from these responses were analysed using the SPSS-X program to provide descriptive statistics for the demographic data, the use of manipulatives and teacher beliefs about mathematics, mathematics learning and mathematics teaching. As well, inferential statistics dealing with relationships between these variables were generated.

Results

This paper reports on some of the baseline data concerning secondary mathematics teachers' demographics and their beliefs about the nature of mathematics, mathematics learning and mathematics teaching.

Demographic data

Of the 249 respondents, 112 (45%) were female. The respondents appear to be a relatively well experienced group, with 64 (26%) of the respondents having in excess of 20 years teaching experience, while 83 (33%), 51 (21%), 42 (17%) had from 11 to 20 years, 6 to 10 years, 1 to 5 years teaching experience respectively. Only 8 respondents (3%) had less than one year of teaching experience.

Fifty-four (22%) of the respondents had been at their current school for more than 10 years, while 89 (36%) and 74 (30%) had taught in their current school for 6 to 10 years or 1 to 5 years respectively. Only 29 (12%) had been in their current school for less than 1 year. The school positions held by the respondents are reported in Table 1.

Table 1 School positions held n=249

Current position	Number	Percentage
Principal	3	1
Leading Teacher	7	3
Head Teacher (Mathematics)	40	16
Classroom Teacher (Mathematics)	193	78
Other	5	2

Two (1%) of the respondents described themselves as two year trained teachers, 13 (5%) as three year trained, 57 (23%) as four year trained with a Bachelor of Education, 154 (62%) as four year trained with a Bachelors degree followed by a Diploma of Education and 21 (8%) as teachers with more than four years of training.

An attempt was made to ascertain where the respondents saw their major teaching focus. They were asked in which of the Year ranges 7 and 8, 9 and 10, 11 and 12, they had mostly taught in the past 10 years. Sixty-two respondents (25%) saw their major emphasis in Years 7 to 10 while 40 (16%) saw the major emphasis of their teaching being in the senior secondary school (Years 11 and 12). Of the remainder, 127 (51%) reported that they had taught across all Years in the last ten years and data were missing for the remaining 20 respondents.

Statements of beliefs - Frequency of teacher responses

Table 2 reports the overall frequency of teacher responses to the 20 statements of beliefs concerning the nature of mathematics and mathematics learning and teaching. Teachers were asked to respond to each belief statement by indicating whether they disagreed with the statement, were undecided about it, or agreed.

Table 2
Statements of belief - Frequency of responses (Percentage of respondents) n=249

Statement of belief	Disagree	Undecided	Agree
Mathematics			
1. Mathematics is computation	88 (35)	40 (16)	111 (45)
2. Mathematics problems given to students should be quickly solvable in a few steps	153 (61)	50 (20)	43 (17)
3. Mathematics is the dynamic searching for order and pattern in the learner's environment	22 (9)	43 (17)	180 (72)
4. Mathematics is no more sequential a subject than any other	171 (69)	36(15)	39 (16)
5. Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking	12 (5)	31 (12)	201 (81)
6. Right answers are much more important in mathematics than the ways in which you get them	215 (86)	15 (6)	15 (6)
Mathematics learning			
7. Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences	12 (5)	30(12)	201 (81)
8. Students are rational decision makers capable of determining for themselves what is right and wrong	90 (36)	95 (38)	60 (24)
9. Mathematics learning is being able to get the right answers quickly	201 (81)	28 (11)	17 (7)
10. Periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process	18 (7)	26 (10)	202 (81)
11. Students are capable of much higher levels of mathematical thought than has been suggested traditionally	51 (21)	107 (43)	87 (35)
12. Being able to memorise facts is critical in mathematics learning	72 (29)	37 (15)	137 (55)
13. Mathematics learning is enhanced by activities which build upon and respect students' experiences	5 (2)	31 (12)	209 (84)
14. Mathematics learning is enhanced by challenge within a supportive environment	2 (1)	13 (5)	231 (93)
Mathematics teaching			
15. Teachers should provide instructional activities which result in problematic situations for learners	6 (2)	38 (15)	202 (81)
16. Teachers or the textbook - not the student - are the authorities for what is right or wrong	149 (60)	49 (20)	46 (19)
17. The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge	60 (24)	45 (18)	140 (56)
18. Teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding	13 (5)	51 (21)	180 (72)
19. Teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their knowledge	30 (12)	58 (23)	157 (63)
20. It is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect	206 (83)	35 (14)	5 (2)

As with questionnaire research in general, a limitation to this study is that all results must be considered in the context that responses to the belief statements are dependent on the interpretations given to them by the respondents.

Beliefs about mathematics

Forty-five percent of the respondents indicated a belief that *mathematics is computation.* In spite of this, 86% disagreed with *right answers are much more important in mathematics than the ways in which you get them.* As well, 72% believe that *mathematics is the dynamic searching for order and pattern in the learner's environment* and 81% believe that *mathematics is a beautiful, creative and useful human endeavour*, perhaps reflecting the fact that most of the

respondents are university trained mathematicians. However, 69% of respondents disagree with the statement that *mathematics is no more sequential a subject than any other*, suggesting a belief about the lock-step nature of so much secondary mathematics teaching and learning.

Beliefs about mathematics learning

Eighty-one per cent of respondents believe that *mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences.* This is supported by 84% of respondents who believe that *mathematics learning is enhanced by activities which build upon and respect students' experiences* and *by challenge within a supportive environment.* Further, 81% of the respondents saw that *periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process.*

Fifty-five per cent of the respondents agree that *being able to memorise facts is critical in mathematics learning.* Only 35% agreed that *students are capable of much higher levels of mathematical thought than has been suggested traditionally.* The large number (43%) of respondents who were undecided about this statement suggests that either the statement itself was not clear or that the teachers were reflecting on the obligatory syllabuses in secondary schools and suggesting that there was not much scope for students to go beyond this. Eighty-one per cent of respondents disagreed that *mathematics learning is being able to get the right answers quickly.*

Of particular interest is the belief that *students are rational decision makers capable of determining for themselves what is right and wrong* where only 24% of respondents agreed with the statement. This would suggest that teachers do not have a high opinion of their students' independent thinking skills in mathematics.

Beliefs about mathematics teaching

The belief that *teachers should provide instructional activities which result in problematic situations for learners* was agreed to by 81% of the respondents with 60% disagreeing that *teachers or textbooks - not the student - are the authorities for what is right or wrong.* The majority (63%) of respondents agreed that *teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their knowledge.* Neither the nature of these norms nor their source was investigated in the survey but both are of importance in determining the nature of the mathematics classroom.

The belief that *the role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge* was agreed to by 56% of the respondents, while 83% of the respondents disagreed with the statement that *it is unnecessary, even damaging, for teachers to tell students if their answers are correct or incorrect.* Even though 72% of respondents agreed that *teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding*, the combination of these results suggests the continuation of the common, although stereotypical, view of secondary mathematics teachers as content oriented, transmission teachers who reluctantly accept that there may be ways to teach mathematics beyond those they have experienced as students.

Linking beliefs

It could reasonably be expected that teachers' beliefs about mathematics, mathematics learning and mathematics teaching would be linked in various ways. To investigate this, a correlational analysis was applied to the beliefs data supplied by the secondary mathematics teachers. From the 180 correlation coefficients generated in this analysis, 53 were significant to at least $p<.05$. Clearly, it is impossible to discuss all of these in this paper. Instead, we concentrate on two apparently contrasting belief statements about mathematics teaching:

17. The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge.

19. Teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their knowledge.

These two items have been chosen for two reasons. Firstly, they are among the items which have the highest number of significant correlations with other items and some of the highest correlation coefficients. Secondly, they reflect the two ends of the transmission / constructivist continuum referred to earlier.

Significant positive correlations were identified between the first of these statements and the following:

2. *Mathematics problems given to students should be quickly solvable in a few steps* (r=.19, p<.005)
9. *Mathematics learning is being able to get the right answers quickly* (r=.19, p<.005)
12. *Being able to memorise facts is critical in mathematics learning* (r=.19, p<.005)
16. *Teachers or the textbook - not the student - are the authorities for what is right or wrong* (r=.15, p<.05)

Significant negative correlations were identified between Statement 17 and the following:

11. *Students are capable of much higher levels of mathematical thought than has been suggested traditionally* (r=-.14, p<.05)
18. *Teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding* (r=.-19, p<.005)
19. *Teachers should negotiate social norms with the students in order to develop a cooperative learning environment in which students can construct their knowledge* (r=-.18, p<.01)

Significant positive correlations were identified between Statement 19 and the following:

5. *Mathematics is a beautiful, creative and useful human endeavour that is both a way of knowing and a way of thinking* (r=.20, p<.005)
7. *Mathematics knowledge is the result of the learner interpreting and organising the information gained from experiences* (r=.15, p<.05)
8. *Students are rational decision makers capable of determining for themselves what is right and wrong* (r=.19, p<.005)
10. *Periods of uncertainty, conflict, confusion, surprise are a significant part of the mathematics learning process* (r=.14, p<.05)
11. *Students are capable of much higher levels of mathematical thought than has been suggested traditionally* (r=.18, p<.005)
13. *Mathematics learning is enhanced by activities which build upon and respect students' experiences* (r=.22, p<.001)
14. *Mathematics learning is enhanced by challenge within a supportive environment* (r=.22, p<.0005)
15. *Teachers should provide instructional activities which result in problematic situations for learners* (r=.26, p<.0001)
18. *Teachers should recognise that what seem like errors and confusions from an adult point of view are students' expressions of their current understanding* (r=.30, p<.0001)

As well, there were significant negative correlations between Statement 19 and the following:

9. *Mathematics learning is being able to get the right answers quickly* (r=-.13, p<.05)
16. *Teachers or the textbook - not the student - are the authorities for what is right or wrong* (r=-.27, p<.0001)
17. *The role of the mathematics teacher is to transmit mathematical knowledge and to verify that learners have received this knowledge* (r=-.18, p<.01)

While some of these correlation coefficients are relatively low, their levels of significance and the sample size suggest that they should be included in discussions of an overall profile of groups of secondary mathematics teachers.

Profiling teachers

The two groups of belief statements identified through the correlational analysis above give two very different profiles of secondary mathematics teachers. The first group might be categorised as a 'transmission' profile with the teachers believing that their role is to transmit and verify reception of knowledge by their students, and that mathematics learning is characterised by memorisation, quick and correct solutions. Further, this group of teachers is at variance with colleagues who espouse the likelihood that students are capable of more than has traditionally been thought and the value of recognising student errors as a part of the

learning process. In many ways, this is a profile of the stereotypical, traditional mathematics teacher.

The second group might be categorised as a 'constructivist' profile with the teachers believing that their role is negotiate the classroom climate so that students are able to construct their own knowledge and that mathematics learning is characterised by the beauty and relevance of mathematics and has its basis in the experiences of the students. 'Errors', challenge, problems, confusion and conflict all have roles in this learning according to this profile and students are seen as rational beings capable of more than they have been traditionally asked to do. Further, this group of teachers is at variance with colleagues who espouse that mathematics learning and teaching is about transmission and verification of knowledge by an outside authority and about getting right answers. This is a profile of a group of teachers, such as those described by Anderson (1996) and Lo, Wheatley, & Smith (1994), espousing 'constructivist' beliefs about mathematics learning and teaching.

Of course, the teachers who might be thought to comprise the two identified groups do not necessarily belong to one group exclusively. For example, 81 of the teachers (33% of the total sample) have indicated that they agree with both items 17 and 19, the key statements in the formation of these groups. This can be reconciled by recalling that any individual teacher will fall along the continuum of which the two identified groups form the end points.

Conclusion

Teachers hold beliefs towards the nature and the learning and teaching of mathematics. These beliefs do influence their teaching. There is evidence from this study that there are many teachers who espouse sets of beliefs which might be described as 'transmission' beliefs and many who espouse sets of beliefs which could be described as 'constructivist'. That teachers at these two ends of the continuum, and all points in between, can come to grips successfully with the current rigid syllabuses and examination systems in place in NSW secondary schools is amazing. How they do this needs to be investigated further through interviews and classroom observations. The researchers have commenced interviewing Head Teachers, Mathematics to seek some answers from the people who are probably most influential in determining the approaches to mathematics learning and teaching in individual schools. This work will be reported later.

As well as seeking data from interviews with teachers, there is support for the view that teachers' reflection on their classroom experiences can shape and influence their beliefs. Further investigation comparing the espoused beliefs of teachers with their enacted beliefs is required. This will require some detailed classroom observation. Particular emphasis needs to be placed on investigating the effect of classroom experiences on the evaluation and reorganisation of teacher beliefs and the effect of this reorganisation on what occurs in the mathematics classroom.

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